FUNCTION SPACES XII

JULY 9 – 14, 2018

KRAKÓW, POLAND

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Kraków

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Invited speakers

1. **Sergei Astashkin**, Samara University, Samara, Russia
2. **Richard Becker**, IMJ, Paris VI, Paris, France
3. **Fernando Cobos**, Universidad Complutense de Madrid, Madrid, Spain
4. **Yunan Cui**, Harbin University of Science and Technology, Harbin, China
5. **Andreas Defant**, University of Oldenburg, Oldenburg, Germany
6. **Loukas Grafakos**, University of Missouri, Columbia, USA
7. **Dorothee Haroske**, University of Rostock, Rostock, Germany
8. **Agnieszka Kałamajska**, University of Warsaw, Warsaw, Poland
9. **Anna Kamińska**, The University of Memphis, Memphis, USA
10. **Jerzy Kąkol**, A. Mickiewicz University, Poznań, Poland
11. **Pierre Gilles Lemarié-Rieusset**, Université d'Évry, Évry, France
12. **Lech Maligranda**, Luleå University of Technology, Luleå, Sweden
13. **Jose Mendoza**, Universidad Complutense de Madrid, Madrid, Spain
14. **Stanisław Prus**, Maria Curie-Skłodowska University, Lublin, Poland
15. **Enrique Alfonso Sánchez Pérez**, Universitat Politècnica de València, Valencia, Spain
16. **Zhongrui Shi**, Shanghai University, Shanghai, China
17. **Lesław Skrzypek**, University of South Florida, Tampa, Florida, USA
18. **Hans Triebel**, University of Jena, Jena, Germany
19. **Michał Wojciechowski**, Polish Academy of Sciences, Warszawa, Poland
20. **Przemysław Wojtaszczyk**, Polish Academy of Sciences, Warszawa, Poland

CONTENTS

PLENARY LECTURES

ABSTRACTS

Extrapolation description of limiting interpolation spaces

Sergey Astashkin

Samara University, Russia

The main aim of this talk is to give a complete characterization of limiting real interpolation spaces using extrapolation theory. It hinges upon the boundedness of some simple operators (e.g. $f \mapsto f(t^2)/t$ or $f \mapsto f(t^{1/2})$ acting on the underlying lattices that are used to control the K-functionals. Reiteration formulae, extending Holmstedts classical reiteration theorem to limiting spaces, are also proved and characterized in this fashion. The resulting theory gives a unified roof to a large body of literature that, using ad-hoc methods, had covered only special cases of the results obtained here. We will concern also with some applications to Matsaev ideals and Grand Lebesgue spaces.

This is a joint work with K. Lykov and M. Milman [1].

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Convex cones and ordered Banach spaces

Richard Becker

IMJ, IMJ, Paris VI, Paris, France

The theory of *ordered Banach spaces* is very general and many problems of analysis may be viewed as problems of this theory by introducing some *convex cone* which defines an ordering. On the other hand, if B is a Banach space, any convex cone $X \subset B$ defines an ordering on B. If $B = X - X$ then X is said to be *generating*. If $0 \leq x \leq y$ implies that $\|x\| \leq C\|y\|$ for some constant C, then X is said to be *normal* (roughly speaking, X is sharp). If the ordering gives a *lattice* structure on B, and if $|x| \leq |y|$ implies $\|x\| \leq \|y\|$, then B is a *Banach lattice*.

Basic results were obtained by G. Krein (1940), V.L. Klee (1955) and T. Ando (1962). After that, the topic was rather dormant during 30 years. But, after this period, new results were obtained.

To every convex cone $X \subset B$ we associate an *index* $i(X)$ which is a generalization of the *cotype index* for cones (if $X = B$ then $i(B)$ is the cotype index of B). This index is involved in various topics. For example:

If T is a linear operator from B to a Banach space F which is p-*summing* on the cone X, with $1 < p < i(X)'$, where $i(X)'$ is the *conjugate* number of $i(X)$, then T is also 1-summing on X.

If T is a *positive* linear operator from a $C(K)$ space to B then T is r-summing for any $r > i(X)$.

If X is a normal cone, with $i(X) > 1$, then $i(X)$ enables us to study whether B contains uniform copies of ℓ_n^r embedded in B, whose basic vectors are contained in X. Namely, $i(X)$ is the maximun of such r.

When B is a Hilbert space then $i(X) = 1$ or $i(X) = 2$.

Finally, some applications are given within the framework of *tensor products*.

Duality for logarithmic interpolation spaces and applications to Besov spaces

Fernando Cobos

Universidad Complutense de Madrid, Spain

Some applications of the ideas of real interpolation have required to consider logarithmic perturbations of the real method $(A_0, A_1)_{\theta,q,\mathbb{A}}$, with

$$\|a\|_{(A_0,A_1)_{\theta,q,\mathbb{A}}} = \left(\int_0^\infty \left(t^{-\theta} \ell(t)^{\mathbb{A}} K(t,a)\right)^q \frac{dt}{t} \right)^{1/q},$$

where $0 < q \leq \infty, \mathbb{A} = (\alpha_0, \alpha_\infty) \in \mathbb{R}^2, \ell(t) = 1 + |\log t|, \ell(t)^{\mathbb{A}} = \ell(t)^{\alpha_0}$ if $0 < t \leq 1$, $\ell(t)^{\mathbb{A}} = \ell(t)^{\alpha_\infty}$ if $1 < t < \infty$, and now not only $0 < \theta < 1$ but also θ can take the values 0 and 1.

When $0 < \theta < 1$ the theory of spaces $(A_0, A_1)_{\theta,q,\mathbb{A}}$ is very similar to the theory of the real method. But if $\theta = 1$ or $\theta = 0$ there are significant differences in the description by means of the J-functional, duality and other subjects.

We will describe duality for these spaces when $\theta = 1$ or 0, paying special attention to the case $0 < q < 1$. We will also apply the abstract results to compute the dual space of Besov spaces of logarithmic smoothness.

The talk is based on results of joint papers with A. Segurado [3], O. Domínguez [2] and B.F. Besoy [1].

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E-mail: cobos@mat.ucm.es

Gâteaux differentiability of w^*-lower semicontinuous convex functions in Banach spaces and some application

Yunan Cui

Department of Mathematics, Harbin University of Science and Technology, Harbin, 150080, China

In this talk, some necessary and sufficient conditions for Gâteaux differentiability of w^*-lower semicontinuous convex functions on X^{**} with values in $\mathbb{R}$ will be given. Moreover, we will also prove that if X^{**} is separable and f is a w^*-lower semicontinuous convex function in X^{**}, then there exists a dense G_δ subset G of X such that f is Gâteaux differentiable at each point of G. Using this general result, some necessary and sufficient conditions for ball-covering property of X^{**} will be given.

E-mail: yunan_cui@aliyun.com

Bohr's phenomenon for functions on the Boolean cube

Andreas Defant

University of Oldenburg, Germany

We study the asymptotic decay of the Fourier spectrum of real functions on the Boolean cube $\{-1,1\}^N$ in the spirit of Bohr's phenomenon from complex analysis. Every such function admits a canonical representation through its Fourier-Walsh expansion $f(x) = \sum_{S\subset\{1,\ldots,N\}} \widehat{f}(S)x^S$, where $x^S = \prod_{k\in S} x_k$. Given a class $\mathcal{F}$ of functions on the Boolean cube $\{-1,1\}^N$, the Boolean radius of $\mathcal{F}$ is defined to be the largest $\rho \geq 0$ such that $\sum_S |\widehat{f}(S)|\rho^{|S|} \leq \|f\|_\infty$ for every $f \in \mathcal{F}$. We indicate the precise asymptotic behaviour of the Boolean radius of several natural subclasses, as e.g. the class of all real functions on $\{-1,1\}^N$, the subclass made of all homogeneous functions or certain threshold functions. Compared with the classical complex situation subtle differences as well as striking parallels occur. Joint work with Mieczysław Mastyło and Antonio Pérez Hernández.

E-mail: defant@mathematik.uni-oldenburg.de

An alternative to Plancherel's criterion for bilinear operators

Loukas Grafakos

University of Missouri Columbia, USA

A well known criterion, based on Placherel's identity, says that a convolution operator

$$L_m(f)(x) = (f * K)(x) = \int_{\mathbb{R}^n} \widehat{f}(\xi)\widehat{K}(\xi)e^{2\pi i x\cdot\xi}\,d\xi$$

is bounded from $L^2(\mathbb{R}^n)$ to itself if and only if the corresponding multiplier $\widehat{K}$, i.e. the Fourier transform of the kernel K, is an L^∞ function. We obtain a similar characterization for bilinear translation-invariant operators of the form

$$T_m(f,g)(x) = \int_{\mathbb{R}^n}\int_{\mathbb{R}^n} \widehat{f}(\xi)\widehat{g}(\eta)m(\xi,\eta)e^{2\pi i x\cdot(\xi+\eta)}\,d\xi\,d\eta$$

that are bounded from $L^2(\mathbb{R}^n)\times L^2(\mathbb{R}^n)$ to $L^1(\mathbb{R}^n)$. Our study encompasses only smooth multipliers m with bounded derivatives and the characterization we obtain is expressed in terms of the Lebesgue integrability of the multiplier. This is joint work with Danqing He and Lenka Slavíková.

E-mail: grafakosl@missouri.edu

Morrey sequence spaces

Dorothee D. Haroske
University of Rostock, Germany

Morrey (function) spaces and, in particular, smoothness spaces of Besov-Morrey or Triebel-Lizorkin-Morrey type were studied in recent years quite intensively and systematically. Decomposition methods like atomic or wavelet characterisations require suitably adapted sequence spaces. This has been done to some extent already. However, based on some discussion at the conference 'Banach Spaces and Operator Theory with Applications' in Poznań in July 2017 we found that Morrey sequence spaces $m_{u,p} = m_{u,p}(\mathbb{Z}^d)$, $0 < p \leq u < \infty$, have been considered almost nowhere. They are defined as natural generalisations of $\ell_p = \ell_p(\mathbb{Z}^d)$ via

$$m_{u,p} = \Bigg\{ \lambda = \{\lambda_k\}_{k\in\mathbb{Z}^d} \subset \mathbb{C} : \\ \|\lambda|m_{u,p}\| = \sup_{j\in\mathbb{N}_0; m\in\mathbb{Z}^d} |Q_{-j,m}|^{\frac{1}{u}-\frac{1}{p}} \Bigg(\sum_{k:\, Q_{0,k}\subset Q_{-j,m}} |\lambda_k|^p \Bigg)^{\frac{1}{p}} < \infty \Bigg\},$$

where $Q_{i,m}$ are dyadic cubes of side length 2^{-i}, $i \in \mathbb{Z}$, $m \in \mathbb{Z}^d$. Clearly, $m_{p,p} = \ell_p$.

We consider some basic features, embedding properties, the pre-dual, a corresponding version of Pitt's compactness theorem, and can further characterise the compactness of embeddings of related finite-dimensional spaces.

This is joint work with Leszek Skrzypczak (Poznań).

E-mail: dorothee.haroske@uni-rostock.de

Compactness and embedding theorems for Sobolev type spaces defined on metric spaces

Agnieszka Kałamajska

University of Warsaw, Warsaw, Poland

Assume that $X = (X, d)$ is a complete metric space equipped with a metric d and two positive complete Borel measures μ and ν. We discuss compactness of embedding from Sobolev - type space defined on X and subordinated to the measure μ, possibly of fractional type, into $L^q(X;\nu)$, where $1 \leq p, q < \infty$. The involved measures need not be doubling. Our results are formulated using covering families and local two weighted Poincaré type inequalities involving measures μ and ν, which are satisfied on certain covering sets $E_i(r)$ and $E_i'(r)$:

$$\left(\int_{E_i(r)} |u - a_{E_i}(u)|^q d\nu\right)^{1/q} \leq C(r) \left(\int_{E_i(r)} G_r(u)^p d\mu\right)^{1/p}$$

whenever $G_r(u)$ belongs to $L^p(X, \mu)$.

We show how to construct such suitable coverings, recovering several classical embedding and trace embedding theorems on domains and fractal sets in $\mathbb{R}^n$, in the weighted and nonweighted setting. Results are obtained together with Jana Björn.

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E-mail: Agnieszka.Kalamajska@mimuw.edu.pl

Abstract Lorentz spaces and Köthe duality

Anna Kamińska

Department of Mathematical Sciences, The University of Memphis, USA

Given a symmetric Banach function space E and a decreasing positive weight w on $I = (0, a)$, $0 < a \leq \infty$, the generalized Lorentz space $\Lambda_{E,w}$ is defined as the symmetrization of the canonical copy E_w of E on the measure space associated with the weight. A class of functions $M_{E,w}$ is similarly defined in the spirit of Marcinkiewicz spaces as the symmetrization of the space $w\,E_w$. Differently as the Lorentz space, which is a Banach function space, the class $M_{E,w}$ does not need to be even a linear space. Let also $Q_{E,w}$ be the smallest fully symmetric Banach function space containing $M_{E,w}$. An investigation of the Köthe duality of these classes is developed that is parallel to preceding works on Orlicz-Lorentz spaces. The Köthe dual of the class $M_{E,w}$ is identified as the Lorentz space $\Lambda_{E',w}$, while the Köthe dual of $\Lambda_{E,w}$ is $Q_{E',w}$. Several characterizations of $Q_{E,w}$ are obtained, one of them states that a function belongs to $Q_{E,w}$ if and only if its level function in Halperin's sense with respect to w, belongs to $M_{E,w}$. These results are applied to a number of concrete Banach function spaces. In particular a new description of the Köthe dual space is provided for the Orlicz-Lorentz space.

Co-author:
Yves Raynaud, Institut de Mathématiques de Jussieu-Paris Rive Gauche, CNRS / UPMC (Univ.Paris 06) / Univ. Paris-Diderot, France, yves.raynaud@upmc.fr

E-mail: kaminska@memphis.edu

On the separable quotient problem for Banach spaces and spaces $C(X)$ of continuous functions

Jerzy Kąkol

Faculty of Mathematics and Computer Science, A. Mickiewicz University, Poznan, Poland

One of the famous unsolved problems of functional analysis asks (Mazur's problem (1932)) if every (infinite-dimensional) Banach space E has an (infinite-dimensional) separable quotient. Many concrete Banach spaces are known to have separable quotient, for example, reflexive Banach spaces, or even weakly compactly generated Banach spaces. Quite recently Agriros, Dodos and Kanellopoulos, proved that every dual Banach space has a separable quotient. On the other hand, V. Rosenthal (independently Lacey) showed that all Banach space $C(X)$ of continuous (real-valued) functions on X have a separable quotient. We provide several useful methods to examine which Banach spaces admit a separable quotient. The talk gathers also quite new results concerning the separable quotient problem for spaces $C_p(X)$ of continuous functions endowed with the pointwise topology. A connection with Efimov compact spaces X will be also discussed.

Optimal spaces for Navier-Stokes equations and other PDEs

Pierre-Gilles Lemarié-Rieusset

LaMME, Univ Evry, CNRS, Université Paris-Saclay, 91025, Evry, France

During the last twenty years, a lot of work has been devoted to the study of NavierStokes equations on the whole space with help of a large variety of function spaces derived from harmonic analysis or interpolation theory. Besov spaces in the late 90's (or BesovMorrey spaces), BMO in the early 2000's and more recently Morrey spaces or singular multiplierr spaces. Many of them have been claimed as being the largest space where to derive existence, uniqueness or regularity theorems. Many of them have turned out not to be as optimal as initially hoped. I shall review some of those spaces from triumph to decay.

E-mail: pierregilles.lemarierieusset@univ-evry.fr

History of the theory of interpolation of operators: 1910–1966

Lech Maligranda

Luleå University of Technology, SE-971 87 Luleå, Sweden

The talk will contain the following parts:

1. Why years from 1910 to 1966 ?;
2. L^p spaces (1910) and Banach spaces (1922);
3. The Riesz–Thorin interpolation theorem (1926, 1939); Why it was and still is useful ?;
4. Banach's interpolation problem 87 from `The Scottish Book`;
5. Orlicz spaces (1932, 1936) and three interpolation theorems of Orlicz (1934, 1954, 1954);
6. The Marcinkiewicz interpolation theorem (1939). Importance of Zygmund (1956, 1960);
7. Generalizations of the Marcinkiewicz interpolation theorem. Lorentz and Marcinkiewicz spaces;
8. The Calderón–Mitjagin interpolation theorem (1965, 1966). Symmetric spaces = rearrangement invariant spaces (1964);
9. The Lions–Peetre real method of interpolation for general Banach spaces = K-method of interpolation (1964). Calderón couples (1966). Quasi-Banach spaces;
10. The Calderón complex methods of interpolation for Banach spaces (1964). The Calderón–Lozanovskii construction for Banach ideal spaces (= Banach lattices).

Of course, I will show many photos. The talk is based on my publications.

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E-mail: *lech.maligranda@ltu.se*

Some results on the Fixed Point Property

José Mendoza

Universidad Complutense de Madrid, Spain

The aim of this talk is to survey some of the most important results related to the fixed point property (FPP) on Banach spaces, and then focus on one open problem on the subject: *do all renormings of* c_0 *fail the (FPP)?* Working on this problem, J. M. Álvaro, P. Cembranos and the speaker have found a sufficient condition for a renorming of c_0 to fail the FPP which is more general than the previously known ones [J. Math. Anal. Appl. 454 (2017), 1106–1113]. This condition as well as some related facts will be analyzed.

E-mail: Jose_Mendoza@mat.ucm.es

Fixed point property and direct sums of Banach spaces

Stanisław Prus

Maria Curie-Skłodowska University, Lublin, Poland

We consider general direct sums of the form $\left(\sum_{i\in I} X_i\right)_E$, where $\{X_i\}_{i\in I}$ is a family of Banach spaces and E is a Banach lattice of functions on the set I. Properties of such sums depend on the lattice E, so we discuss the concepts of uniform monotonicity and order uniform smoothness of Banach lattices and moduli corresponding to these properties.

Next we give an overview of some recent results on the problem under what assumptions direct sums have the fixed point property for a given class of mappings. We focus mainly on the class of nonexpansive maps, i.e., the maps satisfying the Lipschitz condition with constant 1. It is known that many geometric properties imply the fixed point property for nonexpansive mappings. The list of such properties includes uniform nonsquareness, Opial property and García-Falset condition. We discuss coefficients and moduli corresponding to these properties and show how their values for the direct sum $\left(\sum_{i\in I} X_i\right)_E$ can be estimated in terms of the values for the spaces X_i.

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E-mail: stanislaw.prus@umcs.lublin.pl

Integration with respect to vector capacities in Information Sciences

Enrique A. Sánchez Pérez

Instituto Universitario de Matemática Pura y Aplicada, Universitat Politècnica de València, Valencia, Spain

Information measures is an outstanding topic with applications in several research fields, including Computer Science, Econometry and Information Science. Some of these measures are actually defined as non-additive integrals of functions with respect to (non-additive) vector valued capacities. Thus, in this talk we explain several open research lines in which abstract integration techniques play a prominent role. In particular, we will briefly present the following topics:

1) Choquet integration with respect to vector valued capacities and modeling of information measures. Models for economical control processes.

2) Information indexes associated to non-additive vector valued integrals: the Bochner norm, the Pettis norm, the norm of the integral.

3) Applications: Stability of multiple impact indexes from the point of view of integration with respect to non-additive set functions. Probabilistic models for prediction of publication behavior and impact factors.

We will also give an overview of the integration theory underpinning these techniques, focusing on the structure of the spaces of integrable functions involved.

The results presented in this talk have been obtained in collaboration with J.M. Calabuig, O. Delgado, A. Ferrer-Sapena and R. Szwedek.

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Existence of solutions for Kirchhoff type problems in Musielak-Orlicz-Sobolev spaces

Zhongrui Shi

Shanghai University, China

In this paper, we investigate a class of Kirchhoff type problem with Neumann boundary data in Musielak-Orlicz-Sobolev spaces. Using the Musielak-Orlicz theory and Mountain pass theorem, we establish the existence of nontrivial weak solutions which generalizes the existing results.

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E-mail: zshi@shu.edu.cn

Function spaces with dominating mixed smoothness

Hans Triebel

University of Jena, Germany

Sobolev spaces $S^r_pW(\mathbb{R}^n)$ with dominating mixed smoothness normed by

$$\|S^r_pW(\mathbb{R}^n)\| = \sum_{\substack{\alpha\in\mathbb{N}^n_0,\\ 0\le\alpha_j\le r}} \|D^\alpha f\,|L_p(\mathbb{R}^n)\|$$

$r \in \mathbb{N}$, $1 < p < \infty$ and their Besov counterparts $S^r_{p,q}B(\mathbb{R}^n)$, $r > 0$, $1 \le p \le \infty$, $1 \le q \le \infty$ have been introduced by S.M. Nikol'skij in the early 1960s. They, and their counterparts $S^r_pW(Q)$, $S^r_{p,q}B(Q)$ on the cubes $Q = (0,1)^n$ proved to be very effective especially in connection with approximation, sampling, numerical integration etc. One may ask of whether these advantages (compared with their isotropic counterparts) can be preserved when switching from Q to arbitrary (bounded) domains. But for this purpose one needs first some properties for the related spaces in $\mathbb{R}^n$ which are, so far, not available and which will discussed in the talk: (smooth and non–smooth) atoms, pointwise multipliers, homogeneity at the small, localization, fibre-preserving diffeomorphisms.

E-mail: hans.triebel@uni-jena.de

On the Auerbach-Mazur-Ulam problem

Michał Wojciechowski
IMPAN, Warszawa, Poland

In 1935 Auerbach, Mazur and Ulam proved that any centrally symmetric body in R^3 with all two dimensional central sections affinely equivalent to each other is an ellipsoid. This theorem was later generalized to all odd dimensions by Gromov. The proofs are based on the algebraic topology - nonexistence of a non-vanishing vector field tangent to the sphere. This is the reason why in even dimensions the problem is still open - this argument does not work there.

We present a new approach to the problem, that does not use the homological properties of sphere. Under some mild smoothness condition we prove the theorem in 3D using only differential properties of the body. We hope that this approach will work in even dimensions.

Joint work with Bartek Zawalski.

Reduced basis method, Data assimilation and Generalized Sampling

Przemysław Wojtaszczyk

Institute of Mathematics, Polish Academy of Sciences, Poland

1. Reduced basis method. Let $F \subset \mathbb{R}^d$ be a compact set e.g. $F = [0,1]^d$ with $d = 50$ and let

$$D_\mu f = g \quad \mu \in E \tag{1}$$

be a family of uniformly elliptic PDE's (or some other equations). For a given $\mu \in F$ solving (1) is time consuming.

We want to prepare ourselves to do it fast for given μ. Idea of reduced basis method: (end of XX-century) We solve (1) for $\mu_1, \dots, \mu_n$ to get $f_{\mu_1}, \dots, f_{\mu_n}$ and for given μ we approximate the solution

$$f_\mu \sim \sum_{j=1}^{n} a_j f_{\mu_j}.$$

How to find good basis elements?

2. Greedy selection (Maday-Patera-Turinici, 2002)

Let $\mathcal{K} =: \{f_\mu \ : \ \mu \in \Delta\}$ be a compact subset of certain Banach space $\mathcal{X}$ or a Hilbert space $\mathcal{H}$.

We define $\mu_1, \dots \mu_n$ as follows ($f_j = f_{\mu_j}$)

(1) $f_1 = \operatorname{argmax}\{\|f\| \ : \ f \in \mathcal{K}\}$
(2) Given $f_1, \dots, f_n$ we define $V_n = \operatorname{span}\{f_1, \dots, f_n\}$ and put $f_{n+1} = \operatorname{argmax}\{\operatorname{dist}(f, E_n) \ : \ f \in \mathcal{K}\}$

We define $\sigma_n(\mathcal{K}) = \sup_{f \in \mathcal{K}} \operatorname{dist}(f, V_n)$.

Thus we know that

$$\mathcal{K} \subset \bigcap_{j=1} \{f \in \mathcal{X} \ : \ \operatorname{dist}(f, V_n) \leq \sigma_n\} =: \mathcal{K}^{greedy}. \tag{2}$$

This is a classical constructive approximation theory setup: f is smooth if and only if it can be approximated by polynomials, splines, wavelets etc. with certain accuracy.

3. Data assimilation Now suppose we do not know μ but have some data about the solution $f \in \mathcal{K}$, say they are linear functionals

$$l_1, \dots, l_m \in \mathcal{X}^* \text{ and we have } l_j(f) \tag{3}$$

This set is generally intractable so we look at the hopefully easier problem: *Find f in $\mathcal{K}^{greedy}$ satisfying (3).* Usually we do it step by step i.e. we work with the problem $\mathcal{X}$ is a Banach space and $V_1 \subset V_2 \subset \cdots \subset \mathcal{X}$ are finite dimensional subspaces with $\dim V_j = j$ and $\bigcup_j V_j$ dense in $\mathcal{X}$. $l_1, l_2, \dots \subset \mathcal{X}^*$ are linear functionals, we define $M_m(x) = (l_j(x))_{j=1}^m \in \mathbb{R}^m$. Given n and m we want to find a map $\Phi : \mathbb{R}^m \to \mathcal{X}$ such that the element $\Phi(\vec{a}) =: f_{n,m} \in \mathcal{X}$ and $\operatorname{dist}(f_{n,m}, V_n)$ is small and $l_j(f_{n,m}) = \alpha_j$ where $\vec{a} = (\alpha_1, \dots, \alpha_m)$.

4. Generalized Sampling $\mathcal{X}$ is a Banach space and $V_1 \subset V_2 \subset \cdots \subset \mathcal{X}$ are finite dimensional subspaces with $\dim V_j = j$ and $\bigcup_j V_j$ dense in $\mathcal{X}$. $l_1, l_2, \cdots \subset \mathcal{X}^*$ are linear functionals, we define $M_m(x) = (l_j(x))_{j=1}^m \in \mathbb{R}^m$. For given n and m we want to find a map $\Phi : \mathbb{R}^m \to \mathcal{X}$ such that for any $x \in \mathcal{X}$ the element $\Phi(M_m(x)) =: f_{n,m} \in V_n$ and $\|x - f_{n,m}\|_{\mathcal{X}}$ is comparable with $\mathrm{dist}(x, V_n)$.

Clearly the words "small" and "comparable" must be made precise.

Those two problems are very similar and closely related. In [2, 3] we worked out the general approach to those problems. Generalized sampling in case of the Hilbert spaces was extensively studied by Ben Adcock with coauthors, see e.g. [1] and our approach generalises their arguments to the setting of arbitrary Banach space.

In the talk I will describe this scheme and explain some of its applications for questions of classical approximation theory.

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COMMUNICATIONS

ABSTRACTS

Pairs of spaces having the Bishop-Phelps-Bollobás property for operators when the domain is ℓ_∞^n

María D. Acosta

University of Granada, Spain

Bishop-Phelps Theorem states that the set of norm attaining functionals is dense in the (topological) dual of a Banach space. Bollobás showed a "quantitative" version of that result called nowadays the Bishop-Phelps-Bollobás Theorem. He proved that every pair of elements (x_0, x_0^*) in $S_X \times S_{X^*}$ such that $x_0^*(x_0) \sim 1$ can be approximated by another pair (x, x^*) in $S_X \times S_{X^*}$ such that $x^*(x) = 1$. In 2008 it was initiated the study of versions of such result for operators [1]. A pair of Banach spaces (X, Y) has the Bishop-Phelps-Bollobás property for operators (BPBp for short) whenever every pair (x_0, S_0) in $S_X \times S_{L(X,Y)}$ such that $\|S_0(x_0)\| \sim 1$ can be approximated by another pair (x_1, T) in $S_X \times S_{L(X,Y)}$ such that $\|T(x_1)\| = 1$. Here we denote by $L(X, Y)$ the space of bounded and linear operators from X to Y. It is known that the previous property is non trivial. It is an open problem whether or not the pair (c_0, ℓ_1) has the BPBp in the real case. It is known that the pair (ℓ_∞^3, ℓ_1) has that property. We provided a characterization of the Banach spaces Y such that (ℓ_∞^4, Y) has the BPBp. More recently we extended such result to the case where the domain is ℓ_∞^n, where n is any positive integer. As a consequence, we provide examples of classes of spaces Y such that the pair (ℓ_∞^n, Y) has the BPBp for operators.

The results are part of two joint works with J.L. Dávila and M. Soleimani-Mourchehkhorti.

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E-mail: dacosta@ugr.es

Variable exponent Besov-Morrey spaces

Alexandre Almeida

University of Aveiro, Portugal

In this talk we introduce Besov-Morrey spaces with all exponents variable and discuss various fundamental properties, including characterizations in terms of maximal functions, atoms and molecules. These new spaces are introduced from appropriate variable exponent mixed Morrey-sequence spaces which in turn are defined within the framework of semimodular spaces.
This is based on joint work with A. Caetano.

E-mail: jaralmeida@ua.pt

Schauder theory for parabolic equations in variable Hölder spaces

Piotr Michał Bies

Department of Mathematics and Information Sciences, Warsaw University of Technology, Poland

I shall present my results in Schauder theory for parabolic equations for parabolic equations during the talk. I will show main Theorem in general form. I will give main steps of the proof and say something about difficulties, which appears in work with variable exponent. I will tell also, why a study of parabolic equations is different from research of elliptic equations. The elliptic equations in variable Hölder spaces were considered by me in work [1].

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E-mail: biesp@mini.pw.edu.pl

Averaging operators on decreasing or positive functions: equivalence and optimal bounds

Santiago Boza

Universitat Politècnica de Catalunya, Barcelona, Spain

In this talk we will deal with the study of the optimal bounds for the Hardy operator S minus the identity, as well as S and its dual operator S*, for the cases of decreasing, positive or general functions, on the full range $1 \leq p \leq \infty$. In fact, these two kinds of inequalities are shown to be equivalent for the appropriate cone of functions. For $1 \leq p \leq 2$, we prove that these estimates are the same, but for $2 < p < \infty$ they exhibit a completely different behavior. This is a joint work with Javier Soria.

E-mail: santiago.boza@upc.edu

Some function spaces questions arising in problems of diffraction by planar screens

António Caetano

University of Aveiro, Portugal

Recently [1], S. Chandler-Wilde and D. Hewett have proposed a boundary integral equation approach for studying scattering problems involving fractal structures, in particular planar screens which are fractal or have a fractal boundary. This led them to consider, e.g., subspaces of Bessel-potential spaces like

$$H_F^s := \{u \in H^s(\mathbb{R}^n) : \operatorname{supp} u \subset F\}$$

when F is a closed subset of $\mathbb{R}^n$ and

$$\widetilde{H}^s(\Omega) := \overline{\mathcal{D}(\Omega)}^{H^s(\mathbb{R}^n)}$$

when Ω is an open subset of $\mathbb{R}^n$ and, together with A. Moiola, study some properties of such spaces.

As examples of questions of interest in this regard we have the following:

- For which $s \in \mathbb{R}$ and Ω open do we have $\widetilde{H}^s(\Omega) = H^s_{\overline{\Omega}}$?
- For which $s \in \mathbb{R}$ and K compact with empty interior but with positive Lebesgue measure do we have $H^s_K \neq \{0\}$? Or $H^s_K = \{0\}$?
- When is H^t_F dense in H^s_F for F closed and $t > s$?

I shall report on this and also on some answers to which we arrived during our recent collaboration project.

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E-mail: acaetano@ua.pt

Continuity and compactness of the composition operator between distinct Orlicz spaces

Tadeusz Chawziuk

Adam Mickiewicz University, Pozna„, Poland

We consider the composition operator between distinct Orlicz spaces, ie generated by different Young functions over different measure spaces. We state necessary and/or sufficient conditions for the continuity and compactness of this operator under various conditions on the underlying measure spaces, the transformation inducing the composition operator, and/or the generating Young functions. Compactness is tackled through the concept of uniform absolute continuity. The function and sequence case are treated separately.

E-mail: tchawz@gmail.com

Disjoint hypercyclic operators on groups

Chung-Chuan Chen

National Taichung University of Education, Taiwan

About one decade ago, Bernal-Gonález, Bès and Peris introduced the new notion of disjoint hypercyclicity respectively. Since then, disjoint hypercyclicity was studied intensively. In this talk, we will recall this new notion by studying some classic examples of weighted shifts on the integer group, first. Then we subsume these results by providing the characterization for weighted translation operators on locally compact groups to be disjoint hypercyclic.

E-mail: chungchuan@mail.ntcu.edu.tw

Relationships between the best dominated approximation in the sense of the Hardy-Littlewood-Pólya relation and strict K-monotonicity and K-order continuity in symmetric spaces.

Maciej Ciesielski

Poznań University of Technology, Poznań, Poland

Let L^0 be a set of all (equivalence classes of) extended real valued m-measurable functions on $I = [0, \alpha)$, where $0 < \alpha \leq \infty$. For any $x \in L^0$ we denote $x^*(t) = \inf\{\lambda > 0 : m(|x| > \lambda) \leq t\}$, $x^{**}(t) = \frac{1}{t}\int_0^t x^*(s)ds$ for $t > 0$. The *Hardy-Littlewood-Pólya relation* $\prec$ is given for any x, y in $L^1 + L^\infty$ by

$$x \prec y \Leftrightarrow x^{**}(t) \leq y^{**}(t) \text{ for all } t > 0.$$

Let $(E, \|\cdot\|_E)$ be a symmetric space and let $Y \subset X$ be a nonempty subset. For $x \in X$ denote

$$P_Y(x) = \{y \in Y : \|x - y\| = dist(x, Y)\}.$$

Any element $y \in P_Y(x)$ is called a best approximant in Y to x. A nonempty set $Y \subset X$ is called *proximinal* or *set of existence* if $P_Y(x) \neq \emptyset$ for any $x \in X$. A nonempty set Y is said to be a *Chebyshev set* if it is proximinal and $P_Y(x)$ is a singleton for any $x \in E$.

A symmetric space E is said to be *strictly K-monotone* (shortly $E \in (SKM)$) if for any $x, y \in E$ such that $x^* \neq y^*$, $x \prec y$ we have $\|x\|_E < \|y\|_E$. A point $x \in E$ is called a *point of K-order continuity* of E if for any $(x_n) \subset E$ such that $x_n \prec x$ and $x_n^* \to 0$ a.e. we have $\|x_n\|_E \to 0$. A symmetric space E is called *K-order continuous* (shortly $E \in (KOC)$) if every element x of E is a point of K-order continuity.

We discuss a characterization of strict K-monotonicity and K-order continuity in symmetric spaces. We present a connection between strict K-monotonicity, K-order continuity and the best dominated approximation problems with respect to the Hardy-Littlewood-Pólya relation $\prec$. The above results are based on the paper [1].

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E-mail: maciej.ciesielski@put.poznan.pl

Cyclicity on weighted Hardy spaces and applications

Volodymyr Dilnyi

Cracow University of Technology, Poland

Let $H^p_\sigma(\mathbb{C}_+), \sigma \geq 0, 1 \leq p < +\infty$, is the space of analytic functions in the half-plane $\mathbb{C}_+ = \{z : \mathrm{Re} z > 0\}$ for which

$$||f|| := \sup_{-\frac{\pi}{2} < \varphi < \frac{\pi}{2}} \left\{ \int_0^{+\infty} |f(re^{i\varphi})|^p e^{-pr\sigma|\sin\varphi|} dr \right\}^{1/p} < +\infty.$$

For the case $\sigma = 0$ the space $H^p_\sigma(\mathbb{C}_+)$ is the (classical) Hardy space.

A function G is called *cyclic* in $H^p_\sigma(\mathbb{C}_+)$, $p \geq 1$, if $G \in H^p_\sigma(\mathbb{C}_+)$ and the system

$$\{G(z)e^{\tau z} : \tau \leq 0\}$$

is complete in $H^p_\sigma(\mathbb{C}_+)$.

Theorem. *Let $G \in H^2_\sigma(\mathbb{C}_+), \sigma > 0, G \not\equiv 0$. Then G is cyclic in $H^2_\sigma(\mathbb{C}_+)$ if and only if the function G is zero-free in $\mathbb{C}_+$, the singular boundary function of G is an identical constant and*

$$\limsup_{x\to+\infty} \left(\frac{\ln|G(x)|}{x} + \frac{2\sigma}{\pi} \ln x \right) = +\infty.$$

Also, we discus about applications in the signal theory, zeta function theory, convolution equations and others.

E-mail: dilnyiv@gmail.com

s-numbers of general diagonal operators.

Alicja Dota

Poznań University of Technology, Poland

During the last decade there has been a considerable interest in entropy and s-numbers of Sobolev embeddings. This interest has its origin in applications to spectral theory of (pseudo-)differential operators, via the famous Carl-Triebel or Pietscha-Weyla inequality.

Estimating entropy and s-numbers of function spaces embeddings is not an easy task. The first step is usually using the technique of discretization by wavelet bases, atomic or subatomic decompositions i.e., we can reduce the problem to the corresponding problem for suitable sequence spaces. However the resulting sequence spaces, are still quite complicated, often they are of mixed-norm type and/or involve weights. Therefore a further reduction is necessary, which by factorization leads to diagonal operators in ℓ_p-spaces. In most cases are known results for entropy numbers, however a problem optimal s-numbers estimates for diagonal operator is still open.

E-mail: alicja.dota@put.poznan.pl

Some properties of Riesz-Bessel transforms associated with the generalized shift operator

Ismail Ekincioglu

Dumlupinar University, Turkey

In this talk, the higher order Riesz-Bessel transforms related to generalized shift operator is introduced. However, some properties of the generalized shift operator is examined. Then the boundedness of higher order Riesz-Bessel transforms in weighted $L_{p,\gamma}$-spaces is proved.

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E-mail: ismail.ekincioglu@dpu.edu.tr

Product Factorability of Bilinear Maps on Banach Function Spaces

Ezgi Erdoğan
Marmara University, Turkey

In this presentation, we consider the bilinear operators acting in pairs of Banach function spaces and we give a factorization theorem for these maps by a summability condition associated to the product. Our main result establishes that if we consider bilinear maps acting in Banach function spaces that have a factorization through pointwise product, this factorization gives the class of symmetric bilinear operators, that coincide with the so called zero product preserving operators –maps that are zero valued for couples of functions whose pointwise product is zero–. In other words, for a bilinear map $B : X(\mu) \times Y(\mu) \to Z$ the followings imply each other under some requirements

i. B factors through the pointwise product $\odot$
ii. The equality $B(\chi_A, \chi_C) = B(\chi_{A\cap C}, \chi_{A\cup C})$ is satisfied for every $A, C \in \Sigma$.
iii. $f \odot g = 0$ implies $B(f, g) = 0$ for all $(f, g) \in X(\mu) \times Y(\mu)$.

Lastly, we will apply these tools to provide new descriptions of some classes of bilinear integral operators, and to obtain integral representations for abstract classes of bilinear maps satisfying certain domination properties.

This is a joint work with Enrique A. Sánchez Pérez.

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E-mail: ezgi.erdogan@marmara.edu.tr

Estimates of certain localization operators associated with the Riemann-Liouville operator

Soroosh Mohammadi Farsani

Department of Mathematics, Koosha College of Applied Science and Technology, Tehran, Iran

In this work, criteria for the boundedness and compactness of a class of pseudo-differential operators known as time-frequency localization operators are given.

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E-mail: s_mbahman@yahoo.com

Some remarks on geometry of Orlicz-Lorentz spaces

Paweł Foralewski

Faculty of Mathematics and Computer Science, Adam Mickiewicz University, Umultowska 87, 61-614 Poznań, Poland

In this talk some new results concerning geometry of Orlicz-Lorentz spaces will be given.

E-mail: katon@amu.edu.pl

Some properties in Banach lattices defined by classes of linear operators

Jan Fourie

North-West University, South Africa

Classical properties in Banach spaces and Banach lattices, such as the Dunford-Pettis property, DP^*-property, Gelfand-Phillips property, Schur property, etc. are sometimes characterized in terms of certain classes of bounded linear operators. In recent years several variants of these properties (mostly, introducing weaker versions thereof), such as p-Dunford-Pettis, DP^* of order p, p-Schur properties, etc. have been considered. Classes of operators, such as the p-convergent operators, weak* p-convergent operators, p-limited and sequentially limited operators, play important roles in this study. We discuss some results concerning these operators and their applications to the geometrical properties in Banach spaces and Banach lattices, primarily based on the papers mentioned in the references below.

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E-mail: jan.fourie@nwu.ac.za

The universal Banach space with a K-suppression unconditional basis

Joanna Garbulińska-Węgrzyn
Jan Kochanowski University, Kielce, Poland

We apply the categorical method of Fraïssé limits for constructing a universal space $\mathbb{U}_K$ in the class of Banach spaces with a normalized K-suppression unconditional Schauder basis. The universal space constructed by this method has a nice property of extension of almost isometries, which is better than just the standard universality, established in the papers of Pełczyński and Schechtman (who gave a short alternative construction of universal space for class of Banach spaces with an unconditional bases). We also prove that the universal space $\mathbb{U}_K$ is isomorphic to the complementably universal space IU for Banach spaces with unconditional basis, which was constructed by Pełczyński.

On Mazur's property of Banach spaces

Hossein Hosseini Giv

University of Sistan and Baluchestan, Zahedan, Iran

Weak compactness of the closed unit ball of a Banach space is a necessary and sufficient condition for its reflexivity. This statement relates reflexivity, which is a Banach space property defined using a space and its bidual, to compactness, which concerns a Tychonoff space and its Stone–Čech compactification. This motivates us to seek for Banach space arguments in which the bidual is used essentially, and then try to pose similar topological arguments with bidual replaced by the Stone–Čech compactification. In this talk, we discuss a result which asserts that when a Banach space satisfies Mazur's property, its closed unit ball equipped with the weak topology satisfies a similar topological property.

E-mail: hossein.giv@gmail.com, giv@math.usb.ac.ir

On Regular Operators on Banach Lattices

Omer Gok

Yildiz Technical University, Istanbul, Turkey

Let X and Y be Banach lattices. An operator $T : X \to Y$ is called regular if it is difference of two positive operators. $L_r(X,Y)$ denotes the vector space of all regular operators from X into Y. An operator $T : X \to Y$ is called M-weakly compact operator if for every disjoint bounded sequence (x_n) in X, we have $\lim_n \|Tx_n\| = 0.W^r_M(X,Y)$ denotes the regular M-weakly compact operators from X into Y.An operator $T : X \to Y$ is called L-weakly compact if for every disjoint sequence (y_n) in the solid hull of $T(ball(X))$, $\lim_n \|y_n\| = 0$. By $W^r_L(X,Y)$, we denote the set of all regular L-weakly compact operators from X into Y. A Banach lattice X is a KB-space if and only if every increasing norm bounded sequence (x_n) in X converges. A Banach lattice X has b-property if and only if X has an order continuous norm and KB-space. This presentation is devoted to the study of regular operators, M-weakly compact operators and L-weakly compact operators on Banach lattices. In particular, we show that Y has a b-property if and only if $L_r(X,Y)$ has b-property. Also, $W^r_M(X,Y)$ is a KB-space if and only if Y is a KB-space.

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E-mail: gok@yildiz.edu.tr

Singularity analysis of harmonic Bergman kernels

Jiří Jahn

Silesian University in Opava, Mathematics Institute in Opava, Czech Republic

It is known (see [4], [1]) that for the Bergman kernel $B(x, y)$ of a smoothly bounded strictly pseudoconvex domain $\Omega \subset \mathbb{C}^n$ there exist functions $a, b \in C^\infty(\overline{\Omega} \times \overline{\Omega})$ such that for every $x, y \in \Omega$

$$B(x, y) = \frac{a(x, y)}{\rho(x, y)^{n+1}} + b(x, y) \log \rho(x, y),$$

where $\rho(x, y) \in C^\infty(\overline{\Omega} \times \overline{\Omega})$ is such that $\partial\rho(x, y)/\partial y$ and $\partial\rho(x, y)/\partial\overline{x}$ vanish to infinite order and $\rho(x, x) =: \rho(x)$ is a defining function for Ω.

In [3] an analogous result is proved in the context of *harmonic* (instead of holomorphic) Bergman kernels using the calculus of boundary pseudodifferential operators due to L. Boutet de Monvel.

In the talk we discuss an alternative route to the results mentioned above using the theory of holonomic systems of (pseudo)differential equations as exemplified in [5] and [2].

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E-mail: Jiri.Jahn@math.slu.cz

Some fractional integral transforms for functions and dirtributions

Pankaj Jain

South Asian University, New Delhi, India

In this talk, we shall discuss fractional Fourier transform (FFT), fractional sine transform (FST) and fractional cosine transform (FCT). These transforms will be discussed for functions as well as for distributions. Several properties including the differentiation properties of these transforms will be discussed. Also, it is intended to provide L^p-L^q inequality involving generalized convolution related to FFT.

E-mail: pankaj.jain@sau.ac.in

Uniform rotundity and uniform rotundity in every direction of Orlicz function spaces equipped with the p-Amemiya norm

Radosław Kaczmarek

Adam Mickiewicz University in Poznań, Poland

Some results on uniform rotundity and uniform rotundity in every direction of Orlicz function spaces equipped with the p-Amemiya norm ($1 < p < \infty$) will be presented. Recall that these properties are strongly related with the fixed point theory as well as the approximation theory.

E-mail: radekk@amu.edu.pl

The Brown-Halmos Theorem for a Pair of Abstract Hardy Spaces

Oleksiy Karlovych

Universidade Nova de Lisboa, Portugal

Let $H[X]$ and $H[Y]$ be abstract Hardy spaces built upon Banach function spaces X and Y over the unit circle $\mathbb{T}$. We prove an analogue of the Brown-Halmos theorem for Toeplitz operators T_a acting from $H[X]$ to $H[Y]$ under the only assumption that the space X is separable and the Riesz projection P is bounded on the space Y. In particular, we show that $\|a\|_{M(X,Y)} \le \|T_a\|_{\mathcal{B}(H[X],H[Y])} \le \|P\|_{\mathcal{B}(Y)}\|a\|_{M(X,Y)}$, where $M(X,Y)$ is the space of all pointwise multipliers from X to Y. We specify our results to the case of variable Lebesgue spaces $X = L^{p(\cdot)}$ and $Y = L^{q(\cdot)}$ and to the case of Lorentz spaces $X = Y = L^{p,q}(w)$, $1 < p < \infty$, $1 \le q < \infty$ with Muckenhoupt weights $w \in A_p(\mathbb{T})$. This is a joint work with Eugene Shargorodsky (King's College London, UK).

E-mail: oyk@fct.unl.pt

Function spaces with variable exponents

Henning Kempka

University of applied Sciences Jena, Jena, Germany

We introduce Function spaces of Besov- and Triebel-Lizorkin type with all indicees variable. In these spaces not only the smoothness parameter $s(\cdot)$ but also the integrability $p(\cdot)$ as well as the fine index $q(\cdot)$ depend on the space variable $x \in \mathbb{R}^n$.
We present some general results on the variable scales and talk about characterizations with non-smooth atoms, which hold in general for these spaces and provide easy proofs for pointwise multiplier assertions and for intrinsic characterizations on Lipschitz domains.

Maximal function characterizations for Hardy spaces associated to the Laplace-Bessel operator

Cansu Keskin

Dumlupinar University, Turkey

In the classical case, the Hardy space can be also defined either through the boundedness of the maximal operator of the Poisson semigroup or via an atomic decomposition. In this talk, firstly, we introduce the maximal functions associated with the Laplace-Bessel differential operator $\Delta_\nu, (\nu > 0)$. This differential operator is closely connected with the generalized shift operator T^y. Then, we give the definition of the Hardy spaces $H^p_{\Delta_\nu}$ related to the Laplace-Bessel differential operator defined on $\mathbb{R}^n_+$. Finally, we give a characterization of $H^p_{\Delta_\nu}(\mathbb{R}^n_+)$ via using the radial maximal function, the nontangential maximal function and the grand maximal function. For $p > 1$, we demonstrate some relations among L^p_ν Lebesgue space and $H^p_{\Delta_\nu}$ Hardy spaces involving the Laplace-Bessel operator.

This is a joint work with I. Ekincioglu and V. S. Guliyev.

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E-mail: cansu.keskin@dpu.edu.tr, ismail.ekincioglu@dpu.edu.tr

Discrete Morrey spaces and their inclusion properties

Eder Kikianty

University of Pretoria, South Africa

Many operators that are initially studied on Lebesgue spaces $L^p(\mathbb{R}^d)$ have discrete analogues on $\ell^p(\mathbb{Z})$ spaces. Some of these operators have been studied on Morrey spaces $\mathcal{M}^p_q(\mathbb{R}^d)$. In this talk, I will introduce the discrete analogues of Morrey spaces and their generalisations. We provide necessary and sufficient conditions for the inclusion property among these spaces through an estimate for the characteristic sequences. This is a joint work with H. Gunawan (Institut Teknologi Bandung, Indonesia) and Christopher Schwanke (North-West University, South Africa).

E-mail: eder.kikianty@up.ac.za

On approximation processes in Banach spaces defined by a cosine operator function

Andi Kivinukk
(joint work with Anna Saksa)
Tallinn University, Estonia

In our presentation we introduce the Blackman- and Rogosinski-type approximation processes in abstract Banach space setting. The historical roots of these processes go back to W. W. Rogosinski in 1926 ([2]). The given definitions use a cosine operator functions concept ([1]). We prove that in presented setting the Blackman- and Rogosinski-type operators possess the order of approximation, which coincide with results known in trigonometric approximation. Also applications for the different type of approximations will be given.

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E-mail: andik@tlu.ee

Order continuity in abstract Cesaro spaces

Tomasz Kiwerski

Poznań University of Technology, Poznań, Poland

For a Banach ideal space X the abstract Cesàro space CX is the space of all functions f such that $C|f| \in X$, equipped with the norm $\|f\|_{CX} = \|C|f|\|_X$, where C denotes the Cesàro operator

$$C : f \mapsto Cf(x) := \frac{1}{x}\int_0^x f(t)\mathrm{d}t.$$

Here we will focus on considering the abstract Cesàro spaces CX for those function spaces X which are symmetric.

We study the local structure of this spaces in the terms of order continuity We will present a complete characterisation of points of order continuity in abstract Cesàro spaces CX. The main result says that X is order continuous if and only if CX is, under assumption that the Cesàro operator is bounded on X.

The talk is based on a joint paper with Jakub Tomaszewski (Poznań University of Technology).

Mean value property in metric measure space

Antoni Kijowski

Institute of Mathematics of the Polish Academy of Sciences, Poland

We study functions possessing the mean value property in metric measure spaces, [1, 4]. We treat them as a natural counterpart to harmonic functions in this setting. Therefore we examine their properties such as the maximum principle, the Harnack inequality, Lipschitz and Sobolev regularity. Finally, we state necessary and sufficient conditions for function to attain the mean value property expressed via system of PDEs, [3, 5, 6].

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E-mail: akijowski@impan.pl

Symmetrization, factorization and arithmetic of quasi-Banach function spaces

Paweł Kolwicz
Poznań University of Technology, Poland

We investigate relations between symmetrizations of quasi-Banach function spaces and constructions such as Calderón-Lozanovskiĭ spaces, pointwise product spaces and pointwise multipliers. A quasi-normed or normed space $E = (E, \|\cdot\|_E)$ is said to be a *quasi-normed ideal (function) space* or *normed ideal (function) space* on I, where $I = (0, 1)$ or $I = (0,\infty)$ with the Lebesgue measure m, if E is a linear subspace of $L^0(I)$ and satisfies the so-called ideal property, which means that if $y \in E$, $x \in L^0$ and $|x(t)| \le |y(t)|$ for almost all $t \in I$, then $x \in E$ and $\|x\|_E \le \|y\|_E$. If, in addition, E is a complete space, then we say that E is a *quasi-Banach ideal space* or *a Banach ideal space (a quasi-Banach function space or a Banach function space)*, respectively. Let $E = (E, \|\cdot\|_E)$ be a *quasi-normed ideal space* on I. The *symmetrization* $E^{(*)}$ of E is defined as

$$E^{(*)} = \{x \in L^0(I) : x^* \in E\}$$

with the functional $\|x\|_{E(*)} = \|x^*\|_E$. For two quasi-normed ideal spaces E, F on I the *product space* $E \odot F$ is

$$E \odot F = \{u \in L^0(I) : u = x \cdot y \text{ for some } x \in E \text{ and } y \in F\},$$

and for $u \in E \odot F$ we put

$$\|u\|_{E\odot F} = \inf\{\|x\|_E\|y\|_F : u = x \cdot y, x \in E, y \in F\}.$$

The space of (pointwise) *multipliers* $M(E, F)$ is defined as

$$M(E, F) = \{x \in L^0 : xy \in F \text{ for each } y \in E\}$$

with the operator (quasi-)norm

$$\|x\|_{M(E,F)} = \sup_{\|y\|_E=1} \|xy\|_F.$$

We show that under reasonable assumptions the symmetrization commutes with these operations, that is the following equalities are true

$$(E \odot F)^{(*)} = E^{(*)} \odot F^{(*)},$$

$$(E')^{(*)} = (E^{(*)})' \text{and}$$

$$M(E, F)^{(*)} = M(E^{(*)}, F^{(*)})$$

We determine also the spaces of pointwise multipliers between Lorentz spaces. Finally, the above results will be used in proofs of some factorization results. Developed methods may be regarded as an arithmetic of quasi-Banach function spaces.

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E-mail: pawel.kolwicz@put.poznan.pl

Local uniform non-squareness of the Orlicz-Lorentz function spaces

Joanna Kończak

Adam Mickiewicz University in Poznań, Poland

A Banach space $(X, \|\cdot\|)$ is said to be locally uniformly non-square if for any $x \in S(X)$ (the unit sphere of X) there exists $\delta = \delta(x) \in (0,1)$ such that $\min(\|\frac{x-y}{2}\|, \|\frac{x+y}{2}\|) < 1 - \delta$ for any $y \in B(X)$ (the unit ball of X). We will present some results on the local uniform non-squareness of the Orlicz-Lorentz spaces.

E-mail: jkonczak@amu.edu.pl

Weak Derivatives and Sobolev spaces on LCA Groups

Tomasz Kostrzewa

Warsaw University of Technology, Poland

We study Sobolev spaces on LCA groups defined via the Fourier transform [2, 3, 4, 5]. We show that there is an adequate generalisation of differentiation which can be used in theory of Sobolev spaces on LCA groups and we call it an α-weak derivative, where α is a multi-index. Using α-weak derivatives we introduce another definition of Sobolev spaces analogous to the one known from classical analysis. We prove that both the definitions of the Fourier-Sobolev space and the weak derivatives Sobolev space are equivalent under some mild conditions [6].

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E-mail: t.kostrzewa1990@gmail.com

Minimal projection onto certain subspace of $L_p(X \times Y \times Z)$

Michal Kozdęba

Jagiellonian University, Poland

Let X, Y, Z be Banach spaces. We show a formula for a minimal projection from $L_p(X \times Y \times Z)$ onto $L_p(X \times Y) + L_p(X \times Z) + L_p(Y \times Z)$ and its generalization for space $L_p(X_1 \times X_2 \times \ldots \times X_n)$. It is an extension of a result of Cheney and Light who showed a formula for a minimal projection from $L_p(X \times Y)$ onto $L_p(X) + L_p(Y)$.

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E-mail: mkozdeba.ssuj@gmail.com

Weighted L^p conjecture and compactness in L^p spaces

Mateusz Krukowski

Institute of Mathematics, Technical University of Łódź, Poland

The first part of the talk focuses on the weighted L^p-conjecture. A brief revision of spectral theory for commutative Banach algebras enables us to prove the conjecture for locally compact abelian groups. This is an alternative approach to the one known in the literature. Subsequently, the discussion shifts to nilpotent, locally compact groups with the climax being the proof of the weighted L^p-conjecture for these groups.

The second part of the talk begins with an investigation of the Arzelà-Ascoli's theorem and its intricate relationship with the Banach-Alaoglus theorem. Consequently, we put forward the most natural proof (in author's opinion) of the Fréchet-Kolmogorov-Riesz-Weil's theorem for locally compact groups.

E-mail: krukowski.mateusz13@gmail.com

The approximation property for spaces of weighted differentiable functions

Karsten Kruse

Hamburg University of Technology, Germany

For $0 \leq k \leq \infty$ and a family $\mathcal{V}^k := \big((\nu_{j,l,\beta})_{\beta\in\mathbb{N}_0^d,|\beta|\leq l}\big)_{j\in\mathbb{N},0\leq l\leq k}$ of weights on an open set $\Omega \subset \mathbb{R}^d$ we study the space of weighted continuous resp. k-times continuously partially differentiable functions with values in a locally convex (Hausdorff) space $(E,(p_\alpha)_{\alpha\in\mathfrak{A}})$ over a field $\mathbb{K}$ given by

$$\mathcal{CV}^k(\Omega,E) := \{f \in \mathcal{C}^k(\Omega,E) \mid \forall\, j \in \mathbb{N}, l \in \mathbb{N}_0, 0 \leq l \leq k, \alpha \in \mathfrak{A} : |f|_{j,l,\alpha} < \infty\},$$

where

$$|f|_{j,l,\alpha} := \sup_{\substack{z\in\Omega \\ \beta\in\mathbb{N}_0^d,|\beta|\leq l}} p_\alpha\big(\partial^\beta f(z)\big)\nu_{j,l,\beta}(z),$$

and its topological subspace $\mathcal{CV}_0^k(\Omega,E)$ consisting of the functions that vanish with all their derivatives when weighted at infinity.
We show that $\mathcal{CV}_0^k(\Omega,E)$ is isomorphic to the ε-product of L. Schwartz and to the completion of the injective tensor product of $\mathcal{CV}_0^k(\Omega) := \mathcal{CV}_0^k(\Omega,\mathbb{K})$ and E, i.e. $\mathcal{CV}_0^k(\Omega,E) \cong \mathcal{CV}_0^k(\Omega)\varepsilon E \cong \mathcal{CV}_0^k(\Omega)\widehat{\otimes}_\varepsilon E$, if E is complete and the family of weights $\mathcal{V}^k$ fulfills some weak assumptions, which implies that $\mathcal{CV}_0^k(\Omega)$ has the approximation property.
The proof combines the ideas for the case $k = 0$ given in [1, 5.5 Theorem] and for the special case $\mathcal{C}^k(\Omega,E)$ with the topology of uniform convergence of all partial derivatives on compact subsets of Ω from [2, Theorem 44.1].

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E-mail: karsten.kruse@tuhh.de

On some generalization of Cesàro and Copson spaces

Damian Kubiak

Tennessee Technological University, USA

The aim of this project is to define and study basic properties of spaces which generalize the classical Cesàro and Copson spaces on $\mathbb{R}$ to spaces over wider class of measure spaces.

E-mail: dkubiak@tntech.edu

Uniform convergence of trigonometric series

Mateusz Kubiak
University of Zielona Góra, Poland

It is well-known that there is a great number of interesting results in Fourier analysis established by assuming monotonicity of coefficients. The following classical convergence result can be found in many monographs (see [1] and [4], for example).

Theorem. *Suppose that $b_n \geq b_{n+1}$ and $b_n \longrightarrow 0$. Then a necessary and sufficient condition for the uniform convergence of the series*

$$\sum_{n=1}^{\infty} b_n \sin nk$$

is $nb_n \longrightarrow 0$.

This result has been generalized by weakening the monotone condition of the coefficient sequences(for example in [3] and [4]). In this talk we introduce a new class of sequences called $GM(\beta, r, p, q)$, which is the generalization of a class considered by B. Szal in [2]. Moreover, we obtain sufficient and necessary conditions for uniform convergence of trigonometric series with (β, r, p, q) – general monotone coefficients.

The talk is based on the joint work with B. Szal.

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E-mail: M.Kubiak@wmie.uz.zgora.pl

Effective energy integral functionals for thin films on curl-free vector fields in the Orlicz-Sobolev space setting

Włodzimierz Laskowski

West Pomeranian University of Technology, Szczecin

We consider an elastic thin film as a bounded open subset $\omega \subset \mathbb{R}^2$ with Lipschitz boundary. The set $\Omega_\varepsilon := \omega \times (-\frac{\varepsilon}{2}, \frac{\varepsilon}{2}) \subset \mathbb{R}^3$ for a small thickness ε is considered as an elastic cylinder approximate to the film ω. The variational integral functional (the re-scaled kinetic energy of the elastic cylinder Ω_ε) is defined by

$$G_\varepsilon(H) := \begin{cases} \frac{1}{\epsilon} \int_{\Omega_\varepsilon} W(H(x))dx & \text{if } H \in \mathcal{V}_\varepsilon \\ +\infty & \text{otherwise,} \end{cases} \tag{4}$$

where

$$\mathcal{V}_\varepsilon := \{H \in L^M(\Omega_\varepsilon; \mathbb{R}^{3\times 3}) : \text{curl } H = 0 \text{ (distributionally) } \}.$$

The effective energy functional defined on the Orlicz-Sobolev space $W^{1,M}$ over ω is obtained by Γ-convergence and $3D$-$2D$ dimension reduction techniques in the case when the energy density function is cross-quasiconvex. In the case when the energy density function is not cross-quasiconvex we obtained both upper and lower bounds for the Γ-limit.

These results are proved in the case when the energy density function W has the growth prescribed by an Orlicz convex function M. Here M, M^* are assumed to be non-power-growth-type and to satisfy the condition Δ_2^{glob} (that imply the reflexivity of Orlicz and Orlicz-Sobolev spaces generated by M), where M^* denotes the complementary (conjugate) Orlicz N-function of M.

This is a joint work with Hong Thai Nguyen from the University of Szczecin.

E-mail: klesnik@vp.pl

Dimension dependence of factorization problems: one- and two-parameter Hardy spaces

Richard Lechner

Johannes Kepler University Linz, Linz, Austria

We consider the following quantitative factorization problem: We want to factor the identity operator on an n-dimensional (one- or two-parameter) Hardy space through any operator on an $N = N(n)$ dimensional (one- or two-parameter) Hardy space with large diagonal. We improve the best previously known *super-exponential* estimates for N by showing *polynomial* estimates.

Basic properties of Toeplitz and Hankel operators in non-algebraic setting

Karol Leśnik

Poznań University of Technology, Poland

The theory of Toeplitz and Hankel operators is widely investigated and well developed, but mainly in the algebraic setting. In contrast to this, we are interested in a non-algebraic situation, i.e. when these operators act between distinct Hardy spaces. We will present basic properties of such operators and explain how they are related with the problem of factorization of functions and with properties of spaces of pointwise multipliers.

E-mail: klesnik@vp.pl

A note in approximative compactness and midpoint locally k-uniform rotundity in Banach spaces

Chunyan Liu

Shanghai University of Engineering Science, Shanghai, China

In this talk, we prove the following results:

(1) A Banach space X is weak midpoint locally k-uniformly rotund if and only if every closed ball of X is an approximatively weakly compact k-Chebyshev set.

(2) A Banach space X is midpoint locally k-uniformly rotund if and only if every closed ball of X is an approximatively compact k-Chebyshev set.

Pointwise (H, Φ) strong approximation by Fourier series of integrable functions

Włodzimierz Łenski

University of Zielona Góra, Poland

We will present estimations of the generalized strong mean (H, Φ) as an approximation version of the Totik type generalization of the results of J. Marcinkiewicz and A. Zygmund and the classical result of G. H. Hardy and J. E. Littlewood on strong summability of Fourier series of functions from L^1 and from L^Ψ, respectively. As a measures of such approximations we will use the functions constructed, by function Ψ complementary to Φ, on the base of definition of the Gabisonia points G_Ψ and the Lebesgue points L^Ψ.

E-mail: W.Lenski@wmie.uz.zgora.pl

Generalized fractional integrals and central Campanato spaces

Katsuo Matsuoka
College of Economics, Nihon University, Japan

For $0 < \alpha < n$, $d \in \mathbb{N} \cup \{0\}$ and $f \in L^1_{\mathrm{loc}}(\mathbb{R}^n)$, let I_α be a fractional integral, i.e.,

$$I_\alpha f(x) = \int_{\mathbb{R}^n} \frac{f(y)}{|x-y|^{n-\alpha}} \, dy,$$

and $\tilde{I}_{\alpha,d}$ be a generalized fractional integral (of order α), i.e.,

$$\tilde{I}_{\alpha,d}\, f(x) = \int_{\mathbb{R}^n} f(y) \left\{ K_\alpha(x-y) - \left(\sum_{\{l:|l|\le d\}} \frac{x^l}{l!} (D^l K_\alpha)(-y) \right) (1 - \chi_{Q_1}(y)) \right\} dy,$$

where $K_\alpha(x) = \dfrac{1}{|x|^{n-\alpha}}$, D^l is the partial derivative of order $l = (l_1, l_2, \cdots, l_n)$, i.e., $D^l = (\partial/\partial x_1)^{l_1} (\partial/\partial x_2)^{l_2} \cdots (\partial/\partial x_n)^{l_n}$, and χ_{Q_1} is the characteristic function of Q_1, and we put

$$\tilde{I}_\alpha = \tilde{I}_{\alpha,0},$$

which is a modified fractional integral of I_α.

Then, for $0 < \alpha < n$ and $1 \le p < \infty$, the following are known:

- when $-n/p \le \lambda < -\alpha$,
  I_α is well-defined and bounded for $B^{p,\lambda}(\mathbb{R}^n)$;
- when $-n/p \le \lambda < 1 - \alpha$,
  $\tilde{I}_\alpha$ is well-defined and bounded for $\mathrm{CMO}^{p,\lambda}(\mathbb{R}^n)$.

Here, for $1 \le p < \infty$ and $-n/p \le \lambda < \infty$, $B^{p,\lambda}(\mathbb{R}^n)$ is a (non-homogeneous) central Morrey space and $\mathrm{CMO}^{p,\lambda}(\mathbb{R}^n)$ is a (non-homogeneous) λ-central mean oscillation (λ-CMO) space, i.e.,

$$\mathrm{CMO}^{p,\lambda}(\mathbb{R}^n) = \{ f \in L^p_{loc}(\mathbb{R}^n) : \|f\|_{\mathrm{CMO}^{p,\lambda}} < \infty \},$$

where

$$\|f\|_{\mathrm{CMO}^{p,\lambda}} = \sup_{r \ge 1} \frac{1}{r^\lambda} \left(\frac{1}{|Q_r|} \int_{Q_r} |f(y) - f_{Q_r}|^p \, dy \right)^{1/p}$$

and $f_{Q_r} = \frac{1}{|Q_r|} \int_{Q_r} f(y) \, dy$.

In this talk, for the whole of λ such that $-n/p \le \lambda < \infty$, we will extend the results of boundedness of $\tilde{I}_\alpha$ for $\mathrm{CMO}^{p,\lambda}(\mathbb{R}^n)$, i.e., for $-n/p \le \lambda < d + 1 - \alpha$, we will show the boundedness of $\tilde{I}_{\alpha,d}$ for $\mathrm{CMO}^{p,\lambda}(\mathbb{R}^n)$. In order to do this, we use the following function spaces:

- (non-homogeneous) central Campanato space $\Lambda^{(d)}_{p,\lambda}(\mathbb{R}^n)$;
- generalized σ-Lipschitz space $\mathrm{Lip}^{(d)}_{\beta,\sigma}(\mathbb{R}^n)$.

Note: For $r > 0$, $Q_r = \{y \in \mathbb{R}^n : |y| < r\}$ or $Q_r = \{y = (y_1, y_2, \cdots, y_n) \in \mathbb{R}^n : \max_{1\leq i\leq n} |y_i| < r\}$.

E-mail: katsu.m@nihon-u.ac.jp

On monotonic functions from $[0,1]$ into Banach spaces

Artur Michalak

Adam Mickiewicz University, Poznań, Poland

For a function f from the unit interval $[0,1]$ into a Banach space X the oscillation function $d_f : [0,1] \to \mathbb{R} \cup \{\infty\}$ is defined by

$$d_f(t) = \inf_{\delta>0} \sup\{\|f(s) - f(u)\| : s, u \in [0,1], |s-t| \leq \delta, |u-t| \leq \delta\}.$$

For a function $f : [0,1] \to X$ and $\varepsilon > 0$ we put

$$\mathcal{D}(f,\varepsilon) = \{t \in [0,1] : d_f(t) \geq \varepsilon\}.$$

We show that if there exists an increasing function f from $[0,1]$ into a real Banach lattice X such that for some $\varepsilon > 0$ the set $\mathcal{D}(f,\varepsilon)$ is infinite, then X contains a subspace isomorphic to the space $C(\mathcal{D}(f,\varepsilon))$ of all real continuous functions on $\mathcal{D}(f,\varepsilon)$.

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E-mail: michalak@amu.edu.pl

Selected properties of complex symmetric operators

Paweł Mleczko
Paweł Mleczko
Adam Mickiewicz University in Poznań, Poland

In the course of a talk I will discuss interpolation properties of complex symmetric operators on Hilbert spaces and show applications to the study of Toeplitz operators on weighted Hardy–Hilbert spaces of analytic functions on the unit disc. The talk is based on a joint work with Radosław Szwedek from Adam Mickiewicz University in Poznań.

E-mail: pml@amu.edu.pl

Generalized maximal functions in Lorentz spaces

Rza Mustafayev
(joint work with Nevin Bilgiçli)
Department of Mathematics, Faculty of Science, Karamanoglu Mehmetbey University, Turkey

In this work we present the complete characterization of the boundedness of generalized maximal operator

$$M_{\phi,\Lambda^\alpha(b)} f(x) := \sup_{Q \ni x} \frac{\|f\chi_Q\|_{\Lambda^\alpha(b)}}{\phi(|Q|)} \qquad (x \in \mathbb{R}^n),$$

between the classical Lorentz spaces $\Lambda^p(v)$ and $\Lambda^q(w)$, for appropriate functions ϕ, where $0 < p,\, q,\, \alpha < \infty$, v, w, b are weights on $(0,\infty)$ such that $0 < B(t) := \int_0^t b < \infty$, $t > 0$, $B \in \Delta_2$ and $B(t)/t^r$ is quasi-increasing for some $0 < r \le 1$.

E-mail: rzamustafayev@gmail.com

On intertwining non normal operators in a Banach space

Aissa Nasli Bakir

Hassiba Benbouali University of Chlef. Department of Mathematics, Faculty of Exact Sciences and Informatics. Chlef 02000, Algeria

We present some recent results on an extension of the familiar theorem of Fuglede-Putnam Theorem which asserts that each bounded linear operator which intertwines two normal operators defined on a separable complex Hilbert space, then it intertwines their adjoints too. The extension is studied on p-w-hyponormal with dominant operators. Other related results are also shown.

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E-mail: a.nasli@univ-chlef.dz

Well-posedness and Global Attractors for Viscous Fractional Cahn-Hilliard Equations with Memory

Eylem Öztürk

Hacettepe University, Turkey

We examine a viscous Cahn-Hilliard phase-separation model with memory and where the chemical potential possesses a nonlocal fractional Laplacian operator. The existence of global weak solutions is proven using a Galerkin approximation scheme. A continuous dependence estimate provides uniqueness of the weak solutions and also serves to define a precompact pseudometric. This, in addition to the existence of a bounded absorbing set, shows that the associated semigroup of solution operators admits a compact connected global attractor in the weak energy phase space. The minimal assumptions on the nonlinear potential allow for arbitrary polynomial growth.

Joint work with Joseph L. Shomberg.

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E-mail: eyturk1983@gmail.com

On Hardy type inequalities for weighted means

Paweł Pasteczka

Pedagogical University of Cracow, Poland

The aim of this talk is to establish weighted Hardy type inequality in a broad family of means. In other words, for a fixed vector of weights $(\lambda_n)_{n=1}^{\infty}$ and a weighted mean $\mathcal{M}$, we search for the smallest number C such that

$$\sum_{n=1}^{\infty} \lambda_n \mathcal{M}\big((x_1,\ldots,x_n),(\lambda_1,\ldots,\lambda_n)\big) \le C \sum_{n=1}^{\infty} \lambda_n x_n \text{ for all admissible } x.$$

The main results provide a definite answer in the case when $\mathcal{M}$ is monotone and satisfies the weighted counterpart of the Kedlaya inequality. In particular, if $\mathcal{M}$ is symmetric, Jensen-concave, and the sequence $\big(\frac{\lambda_n}{\lambda_1+\cdots+\lambda_n}\big)$ is nonincreasing. In addition, it is proved that if $\mathcal{M}$ is a symmetric and monotone mean, then the biggest possible weighted Hardy constant is achieved if λ is the constant vector.

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E-mail: pawel.pasteczka@up.krakow.pl

Banach spaces with an unconditional basis and a small family of bounded operators

Anna Pelczar-Barwacz

Jagiellonian University, Poland

I will discuss some results on Banach spaces with an unconditional basis and a small family of isomorphisms, namely in which no two disjointly supported block spaces are isomorphic (so-called tight by support spaces). In particular in such spaces no two isomorphic infinitely dimensional subspaces form a direct sum. I will present also the answer to the question of W.T. Gowers, giving an example of a bounded operator on a subspace of Gowers unconditional space (the canonical example of a tight by support Banach space) which is not a strictly singular perturbation of a restriction of a diagonal operator.

The talk is based on a joint paper with Antonis Manoussakis.

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E-mail: anna.pelczar@im.uj.edu.pl

Interpolation of Morrey spaces for general parameters

Yoshihiro Sawano

Tokyo Metropolitan University Japan

Let $1 \leq q \leq p < \infty$. Define the *Morrey norm* $\| \star \|_{\mathcal{M}^p_q}$ by

$$\|f\|_{\mathcal{M}^p_q} \equiv \sup \left\{ |Q|^{\frac{1}{p}-\frac{1}{q}} \|f\|_{L^q(Q)} \ : \ Q \text{ is a cube in } \mathbb{R}^n \right\}$$

for a measurable function f. The *Morrey space* $\mathcal{M}^p_q(\mathbb{R}^n)$ is the set of all the measurable functions f for which $\|f\|_{\mathcal{M}^p_q}$ is finite.

We are interested in the interpolation $[\mathcal{M}^{p_0}_{q_0}, \mathcal{M}^{p_1}_{q_1}]_\theta$ of Morrey spaces. If $\dfrac{q_0}{p_0} = \dfrac{q_1}{p_1}$, then Lemarié-Rieusset, Dachun Yang, Wen Yuan and Winfried Sickel obtained characterizations. Here we are interested in the case $\dfrac{q_0}{p_0} \neq \dfrac{q_1}{p_1}$. This is a joint work with Mieczysław Mastyło.

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E-mail:yoshihiro-sawano@celery.ocn.ne.jp

s-Numbers Ideals of Bilinear Operators

Eduardo Brandani da Silva

Maringa State University, Brazil

The theory of s-numbers of linear bounded operators among Banach spaces was introduced and studied by Pietsch, see [2]. It plays a fundamental role in the theory of operators and the local theory of Banach spaces and it is a powerful tool in the study of eigenvalue distribution of operators in Banach spaces. For multilinear operators, a theory of quasi s-numbers was developed in [1].

Operators ideals were introduced and extensively studied in [3]. This theory gave origin to several papers and books. Operator ideals are fundamental in Functional Analysis and related areas. In this paper we introduce and study quasi s-numbers ideals of bilinear operators among Banach spaces. The relationships among bilinear variants of linear properties and analogous theorems which are well-known in the linear case, are stated and proved.

It shall be noted that whereas the work is based on some ideas from the theory of s-numbers ideals of bounded linear operators, some proofs may be extended from the linear case to the bilinear operators and other require new ideas and methods.

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E-mail: ebsilva@uem.br

Extension of Sawyer's duality to grand Lebesgue spaces

Monika Singh
University of Delhi, India

For L^p-cone of non negative non increasing functions, the following is known as the *Sawyer's duality principle*:

Let $1 < p < \infty$, g *be a non negative measurable function defined on* $(0, \infty)$ *and* w *be a locally integrable weight. Then*

$$\sup_{0 \leq f \downarrow} \frac{\int_0^\infty f(x)g(x)dx}{\left(\int_0^\infty f^p(x)w(x)dx\right)^{1/p}} \approx \left(\int_0^\infty \left(\int_x^\infty \frac{g(t)}{\int_0^t w(s)ds} dt\right)^{p'} w(x)dx\right)^{1/p'}$$

$$\approx \left(\int_0^\infty \left(\int_0^x g(t)dt\right)^{p'} \frac{w(x)}{\left(\int_0^x w(s)ds\right)^{p'}} dx\right)^{1/p'} + \frac{\int_0^\infty g(x)dx}{\left(\int_0^\infty w(x)dx\right)^{1/p}},$$

where the symbol $\approx$ *means that the ratio's of left and right hand sides is bounded between two positive constants depending only on p (and not on w or, g).*

In this communication, our aim is to talk about the Sawyer's duality in the framework of grand Lebesgue spaces $L^{p)}$. Further, its application, in the context of the boundedness of classical Hardy operators on $L^{p)}$ will be given.

E-mail: monikasingh@lsr.du.ac.in

The Fourier Transform in Weighted Lebesgue and Lorentz Spaces

Gord Sinnamon

University of Western Ontario, Canada

I will discuss rearrangement techniques (a.k.a. interpolation techniques) in the study of weighted inequalities for the Fourier Transform.

• A short survey of Lebesgue space results from 1987-2003 leads to a re-opening and resolution of a case believed solved.

• Rearrangement techniques in the Lorentz space case reveal that the Fourier Transform is a "worse case" operator of its boundedness type.

• Bootstrapping gives a method of applying the theory of positive (convolution) operators to tailor rearrangement results to the Fourier Transform specifically.

Some results are joint work with Javad Rastegari.

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E-mail: sinnamon@uwo.ca

Dominated operators, absolutely summing operators and the strict topology

Juliusz Stochmal

University of Zielona Góra, Poland

In the paper [3] Nowak has developed the theory of continuous linear operators on the space $C_b(X,E)$ of bounded continuous functions $f : X \to E$, where X is a completely regular Hausdorff space and E is a Banach space. Then the space $C_b(X,E)$ is equipped with the strict topology β. For X being a locally compact space β coincides with the original strict topology that was introduced in 1958 by Buck [1]. The Riesz Representation Theorem for continuous linear operators $T : C_b(X,E) \to F$ was obtained, where F is a Banach space.

We present results concerning two classes of continuous linear operators on the space $C_b(X,E)$, i.e., dominated and absolutely summing (see [2, §19], [5]). We characterize dominated operators $T : C_b(X,E) \to F$ in terms of their representing measures $m : \mathcal{B}o \to \mathcal{L}(E, F'')$. In particular, we derive an integral representation of dominated operators $T : C_b(X,E) \to F$ with respect to the variation $|m|$ of its representing measure m. Moreover, we show that every absolutely summing operator on $C_b(X,E)$ is dominated.

The talk is based on the paper [4].

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E-mail: juliusz.stochmal@gmail.com

Restrictions of higher derivatives of the Fourier transform to the sphere

Dmitriy Stolyarov

St. Petersburg Department of Steklov Institute & St. Petersburg State University, Russia

Let S^{d-1} be the unit sphere in $\mathbb{R}^d$, $d \geq 2$. The classical Tomas–Stein says that the inequality

$$\big\|\hat{f}\,|_{S^{d-1}}\,\big\|_{L_2(S^{d-1})} \lesssim \|f\|_{L_p(\mathbb{R}^d)}$$

holds true iff $p \in [1, \frac{2d+2}{d+3}]$. Clearly, a similar estimate for a higher derivative of $\hat{f}$ in terms of $\|f\|_{L_p}$ is impossible. We will discuss additional translation invariant conditions on f that make such bounds possible.

Uniform convergence of double sine series

Bogdan Szal

University of Zielona Góra, Poland

It is well known that in Fourier analysis there is a great number of interesting results established by assuming monotonicity of coefficients. The following theorem gives necessary and sufficient conditions for the uniform regular convergence of double sine series.

Theorem. ([5]) *If* $\{c_{jk}\}_{j,k=1}^{\infty} \subset R_+$ *is a monotonically decreasing double sequence, then the series*

$$\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} c_{jk} \sin jx \sin ky$$

is uniformly regularly convergent in (x, y) *if and only if*

$$jkc_{jk} \to 0 \qquad \text{as} \qquad j+k \to \infty.$$

This result was generalized by Kórus and Móricz [2] and by Kórus [3] (and also by Leindler [2]) by weakening the monotone condition of the coefficient sequences. Namely, they have defined new classes of double sequences to obtain those generalizations. In this talk we introduce new larger classes of double sequences and give sufficient conditions for the uniformity of the regular convergence of double sine series with the coefficient sequences belonging to these classes. We present also necessary conditions in the case when the coefficients of double sine series are non-negative. Presented results come from the paper [1].

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E-mail: B.Szal@wmie.uz.zgora.pl

ODE-determined variable L_p spaces

Jarno Talponen
University of Eastern Finland, Joensuu, Finland

In this talk we recall the intuition and some basic facts of the recent ODE approach to defining varying Lebesgue norms which was introduced by the speaker.

In the most recent paper in this line of inquiry we study decompositions of Nakano type varying exponent Lebesgue norms and spaces. These function spaces are represented here in a natural way as tractable varying ℓ^p type sums of projection bands. The main results involve embedding the varying Lebesgue spaces, e.g. of Musielak-Orlicz-Nakano type, to such sums, as well as the corresponding isomorphism constants. The main tool applied here is an equivalent variable Lebesgue norm which is defined by an ODE natural for the purpose. We also discuss the effect of transformations changing the ordering of the unit interval on the values of the ODE-determined norm.

A general theory of introducing norms in Orlicz spaces

Marek Wisła

Adam Mickiewicz University in Poznań, Poland

On the space of measurable functions W. Orlicz [3] and W. A. Luxemburg [2] introduced norms that have been investigated till today. In the year 2000 Hudzik and Maligranda [1] proposed to investigate the family of so-called p-Amemiya norms ($1 \leq p \leq \infty$) that cover both classical norms: the Orlicz one ($p = 1$) and the Luxemburg one ($p = \infty$). During the lecture it will be presented the general theory of constructing the norms on Orlicz spaces by use of the composition of the modular $I_\Phi(x)$ and the so-called outer function s. This theory leads to a very wide class of norms and opens a brand new perspective in the theory of Orlicz spaces. The lecture will end with a theorem establishing the Köthe dual space to an Orlicz space equipped with the generalized norm.

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E-mail: marek.wisla@amu.edu.pl

Strong unique minimal projections onto hyperplanes

Paweł Wójcik

Pedagogical University of Cracow, Polnad

We discuss some results concerning the problem of minimal projections and extensions. Let X be a reflexive Banach space and let Y be a closed subspace of X of codimension one. Let W be a finite-dimensional Banach space. We present a new sufficient condition under which any minimal extension $E \in \mathcal{L}(X, W)$ of an operator $A \in \mathcal{L}(Y, W)$ is strongly unique. In this report we show (in some circumstances) that a minimal projection from X onto Y is a strongly unique minimal projection.

E-mail: pawel.wojcik@up.krakow.pl

Dugundji-type extensions of some classes of Baire-alpha functions

Marek Wójtowicz

Kazimierz Wielki University in Bydgoszcz, Poland

Let A be a nonempty subset of a Hausdorff space X, and let α be an ordinal number $< \omega_1$. By $\mathcal{B}_\alpha(X)$ we denote the space of all real functions $X \to R$ of Baire-class-α. $\mathcal{F}(A)$ is the set of functions $A \to R$ with a property $\mathcal{F}$ such that $\mathcal{F}(A)$ is a linear subspace of $\mathcal{B}_\alpha(X)$.

We prove Borsuk-Dugundji-type extension theorems: we give an explicit form of a linear extension operator $T : \mathcal{F}(A) \to \mathcal{F}(X)$, where A is an F_σ and G_δ-subset of a normal space X. Our results apply for $\mathcal{F} =$ *to be piecewise continuous*, and $\mathcal{F} =$ *to be of Baire class* α. We show that T restricted to the subspace of bounded functions from $\mathcal{F}(A)$ (endowed with the supremum norm) is a positive isometry. In particular, this solves partially an extension problem set in 2005 by Kalenda and Spurný [1].

Moreover, by the use of an affine (yet, non-linear) extension operator $\mathcal{B}_\alpha(A) \to \mathcal{B}_\alpha(X)$ we give an explicit form of extensions preserving bounds of Baire-alpha functions.

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E-mail: mwojt@ukw.edu.pl

On Dunford-Pettis-type functions

Elroy Zeekoei

North-West University, South Africa

The purpose of this talk is to introduce the notion of the so-called p-convergent functions. We also introduce the notions of the Dunford-Pettis* property of order p and relate it to the p-convergent operators and use these notions to characterise the p-convergent functions.

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E-mail: elroy.zeekoei@nwu.ac.za

Fixed point theorems for (L)-type mappings in complete CAT(0) spaces

Jing Zhou

Shanghai University of Engineering Science, Shanghai, China

In this paper, fixed point properties for a class of more generalized nonexpansive mappings called (L)-type mappings are studied in geodesic spaces. Existence of fixed point theorem, demiclosed principle, common fixed point theorem of single-valued and set-valued are obtained in the third section. Moreover, in the last section, *Delta*-convergence and strong convergence theorems for (L)-type mappings are proved. Our results extend the fixed point results of Suzukis results in 2008 and Llorens-Fusters results in 2011.

LIST OF PARTICIPANTS

María Dolores ACOSTA Análisis Matemático Granada, Granada, SPAIN
Asuman AKSOY Mathematics, Claremont McKenna College, Claremont, UNITED STATES
Alexandre ALMEIDA Department of Mathematics, University of Aveiro, Aveiro, PORTUGAL
Sergei ASTASHKIN Mathematical Department, Samara University, Samara, RUSSIAN FEDERATION
Mojtaba BAKHERAD Faculty of Mathematics, Department of Mathematics, University of Sistan and Baluchestan, Zahedan, IRAN
Sudeshna BASU Mathematics, George Washington University, Washington DC, UNITED STATES
Deepika BAWEJA NPSEI, Applied Sciences, NPSEI, Pithoragarh, INDIA
Richard BECKER IMJ, IMJ, Paris VI, Paris, FRANCE
Piotr BIES Department of Mathematics and Information Sciences, University of Warsaw Technology, Warsaw, POLAND
Fatiha BOULAHIA Department of Mathematics, Faculty of Exact Sciences, University Abderrahmane Mira-Bejaia, Bejaia, ALGERIA
Santiago BOZA EETAC, Department of Mathematics, Universitat Politècnica de Catalunya, Castelldefels (Barcelona), SPAIN
António CAETANO Department of Mathematics, University of Aveiro, Aveiro, PORTUGAL
Tadeusz CHAWZIUK Mathematics and Computer Science, Adam Mickiewicz University, Poznań, POLAND
Chung-Chuan CHEN Department of Mathematics Education, National Taichung University of Education, Taichung, TAIWAN
Lili CHEN Department of Mathematics, Harbin University of Science and Technology, Harbin, CHINA
Qingjin CHENG School of Mathematical Sciences, Xiam University, Xiamen, CHINA
Elmiloud CHIL FST, Department of Mathematic, University of Tunis Elmanar, Elmanar Tunis, TUNISIA
Maciej CIESIELSKI Institute of Mathematics, Faculty of Electrical Engineering, Poznań University of Technology, Poznań, POLAND
Fernando COBOS Departamento de Análisis Matemático y Matemática Aplicada, Universidad Complutense de Madrid, Madrid, SPAIN
YunAn CUI School of Science, Department of Mathematics, Harbin University of Science and Technology, Harbin, CHINA
Andreas DEFANT Mathematics, University of Oldenburg, Oldenburg, GERMANY
Beata DERĘGOWSKA Department of Mathematics, Pedagogical University of Cracow, POLAND
Fatih DERINGOZ Mathematics, Ahi Evran University, Kirsehir, TURKEY

Volodymyr DILNYI Institute of Mathematics, WFMI, Cracow University of Technology, POLAND

Alicja DOTA Institute of Mathematics, Faculty of Electrical Engineering, Poznań University of Technology, Poznań, POLAND

Ismail EKINCIOGLU Departments of Mathematics / Art and Science Faculty, Dumlupınar University, Kutahya/TURKEY, TURKEY

Ezgi ERDOĞAN Mathematics, Faculty of Arts and Sciences, Marmara University, Istanbul, TURKEY

Soroosh Mohammadi FARSANI Department of Mathematics, Koosha College of Applied Science and Technology, Tehran, IRAN

Blanca FERNÁNDEZ BESOY Departamento de Análisis Matemático y Matemática Aplicada, Universidad Complutense de Madrid, Madrid, SPAIN

Luz Maria FERNANDEZ-CABRERA Departamento de Analisis Matematico y Matematica Aplicada, Facultad de Matematicas, Universidad Complutense de Madrid, Madrid, SPAIN

Paweł FORALEWSKI Faculty of Mathematics and Computer Science, Adam Mickiewicz University in Poznań, Poznań, POLAND

Jan FOURIE NWU, School of Mathematics and Statistics, North-West University, Potchefstroom, SOUTH AFRICA

Joanna GARBULIŃSKA-WĘGRZYN Institute of Mathematics, The Faculty of Mathematics and Natural Science, Jan Kochanowski University, Kielce, POLAND

Hossein Hosseini GIV Department of Mathematics, University of Sistan and Baluchestan, Zahedan, IRAN

Omer GOK Mathematics Department/ Faculty of Arts and Sciences, Yildiz Technical University, Istanbul, TURKEY

Loukas GRAFAKOS Mathematics, University of Missouri, Columbia, UNITED STATES

Jawad H'MICHANE Sciences Faculty, Departement of Mathematics, Moulay Ismail, Meknes, MOROCCO

Dorothee HAROSKE Institute of Mathematics, Faculty of Mathematics and Natural Sciences, University of Rostock, Rostock, GERMANY

Henryk HUDZIK 1) Faculty of Economics and Information Technology Płock and 2) Faculty of Mathematics and Computer Science Poznań, 1)The State University of Applied Sciences in Płock and 2) Adam Mickiewicz University in Poznań, Poznań, POLAND

Jiří JAHN Mathematical Institute in Opava, Silesian University in Opava, Opava, CZECH REPUBLIC

Pankaj JAIN Department of Mathematics, South Asian University, New Delhi, INDIA

Aref JERIBI Faculty of Sciences of Sfax, Mathematics, University of Sfax, Sfax, TUNISIA

Radosław KACZMAREK Faculty of Mathematics and Computer Science, Adam Mickiewicz University in Poznań, Poznań, POLAND

Agnieszka KAŁAMAJSKA Institute of Mathematics, Faculty of Mathematics, Informatics and Mechanics, University of Warsaw, Warsaw, POLAND

Anna KAMINSKA Math Sciences, The University of Memphis, Memphis, UNITED STATES

Oleksiy KARLOVYCH Faculdade de Ceincias e Tecnologia, Departamento de Matematica, Universidade Nova de Lisboa, Caparica, PORTUGAL

Jerzy KĄKOL Faculty of Mathematics and Computer Science, A. Mickiewicz University, Poznań, POLAND

Henning KEMPKA Departement of Fundamental Dciences, University of applied Sciences Jena, Jena, GERMANY

Cansu KESKIN Departments of Mathematics / Art and Science Faculty, Dumlupınar University, Kütahya, TURKEY

Muhammad Ahsan KHAN ASSMS, Mathematics, GCU Lahore, Lahore, PAKISTAN

Eder KIKIANTY Mathematics and Applied Mathematics, University of Pretoria, Pretoria, SOUTH AFRICA

Andi KIVINUKK Department of Mathematics, Tallinn University, Tallinn, ESTONIA

Tomasz KIWERSKI Institute of Mathematics, Faculty of Electrical Engineering, Poznań University of Technology, Poznań, POLAND

Paweł KOLWICZ Institute of Mathematics, Faculty of Electrical Engineering, Poznań University of Technology, Poznań, POLAND

Joanna KOŃCZAK Faculty of Mathematics and Computer Science, Adam Mickiewicz University in Poznań, Poznań, POLAND

Marta KOSEK Institute of Mathematics, Faculty of Mathematics and Computer Science, Jagiellonian University, Kraków, POLAND

Tomasz KOSTRZEWA Department of Mathematics and Information Sciences, Warsaw University of Technology, Warsaw, POLAND

Michał KOZDĘBA Institute of Mathematics, Department of Mathematics and Computer Science, Jagiellonian University, Kraków, POLAND

Mateusz KRUKOWSKI Institute of Mathematics, Politechnika Łódzka, Łódź, POLAND

Karsten KRUSE Institut für Mathematik, Elektrotechnik, Informatik und Mathematik, Hamburg University of Technology Hamburg GERMANY

Damian KUBIAK Mathematics Department, Tennessee Technological University, Cookeville, UNITED STATES

Mateusz KUBIAK Faculty of Mathematics, Computer Science and Econometrics, University of Zielona Góra, Zielona Góra POLAND

Richard LECHNER Institute of Analysis, Functional Analysis, Johannes Kepler University Linz, Linz, AUSTRIA

Pierre Gilles LEMARIÉ-RIEUSSET LaMME, Département de Mathématiques, Université d'Évry, Évry, FRANCE
Karol LESNIK Institute of Mathematics, Poznań University of Technology, Poznań, POLAND
Barbara LEWANDOWSKA Faculty of Mathematics and Computer Science, Jagiellonian University, Kraków, POLAND
Grzegorz LEWICKI Institute of Mathematics, Department of Mathematics and Computer Science, Jagiellonian University, Kraków, POLAND
Chunyan LIU School of Mathematics, Physics and Statistics, Shanghai University of Engineering Science, Shanghai CHINA
Włodzimierz {LENSKI Faculty of Mathematics, Computer Science and Econometrics, University of Zielona Góra, Zielona Góra, POLAND
Lech MALIGRANDA Department of Engineering Sciences and Mathematics, Luleå University of Technology, Luleå, SWEDEN
Hosseini MALIHEH Department of Mathematics, K. N. Toosi University of Technology, Tehran, IRAN
Mieczysław MASTYŁO Faculty of Mathematics and Computer Science, Adam Mickiewicz University, POLAND,
Katsuo MATSUOKA College of Economics, Nihon University, Tokyo, JAPAN
Joanna MEISSNER Faculty of Applied Mathematics, AGH University of Science and Technology, Kraków, POLAND
José MENDOZA Facultad de Matemáticas, Universidad Complutense de Madrid, Madrid, SPAIN
Artur MICHALAK Faculty of Mathematics and Computer Science, Adam Mickiewicz University, Poznań, Poznań, POLAND
Paweł MLECZKO Faculty of Mathematics and Computer Science, Adam Mickiewicz University in Poznań, Poznań, POLAND
Rza MUSTAFAYEV Department of Mathematics / Faculty of Science, Karamanoglu Mehmetbey University, Karaman, TURKEY
Aissa NASLI BAKIR Department of Mathematics / Faculty of Exact Sciences and Informatics, Hassiba Benbouali University of Chlef, Chlef, ALGERIA
Per NILSSON Private, SWEDEN
Marian NOWAK Faculty of Mathematics, Computer Science and Econometrics, University of Zielona Góra, Zielona Góra, POLAND
Eylem ÖZTÜRK Mathematics Dept. Science Fac., Hacettepe University, Ankara, TURKEY
Paweł PASTECZKA Department of Mathematics, Pedagogical University of Cracow, Kraków, POLAND
Anna PELCZAR-BARWACZ Institute of Mathematics, Faculty of Mathematics and Computer Science, Jagiellonian University, Kraków, POLAND

Stanisław PRUS Institute of Mathematics, Faculty of Mathematics, Physics and Computer Science, Maria Curie-Skłodowska University, Lublin, POLAND
Ali RAZA Abdus Salam School of Mathematical Sciences, Mathematics, GC University Lahore, Lahore, PAKISTAN
Mark ROELANDS North-West University, Potchefstroom, SOUTH AFRICA
Hadi ROOPAEI Mathematics, Vali-e-Asr University of Rafsanjan, IRAN
Michał RZECZKOWSKI Faculty of Mathematics and Computer Science, Adam Mickiewicz University, Poznań, POLAND
Enrique A. SÁNCHEZ PÉREZ Instituto Universitario de Matemática Pura y Aplicada, Universitat Politècnica de València, Valencia, SPAIN
Yoshihiro SAWANO Department of Mathematics, Tokyo Metropolitan University, Hachioji, JAPAN
Zhongrui SHI Mathematics, Shanghai University, Shanghai, CHINA
Eduardo Brandani da SILVA Department of Mathematics, Maringa State University, Maringa, BRAZIL
Arun Pal SINGH Dyal Singh College (University of Delhi), Lodhi Road, Mathematics, University of Delhi New Delhi - 110 003, INDIA
Monika SINGH Lady Shri Ram College for Women (University of Delhi), Lajpat Nagar, Mathematics, University of Delhi New Delhi - 110 024, INDIA
Gord SINNAMON Mathematics, University of Western Ontario, London, CANADA
Leszek SKRZYPCZAK Faculty of Mathematics and Computer Science, Adam Mickiewicz University, Poznań, POLAND
Lesław SKRZYPEK Department of Mathematics and Statistics, University of South Florida, Tampa, Florida, UNITED STATES
Juliusz STOCHMAL Faculty of Mathematics, Computer Science and Econometrics, University of Zielona Góra, Zielona Góra, POLAND
Dmitriy STOLYAROV St. Petersburg Department of Steklov Mathematical Institute, St. Petersburg, RUSSIAN FEDERATION,
Bogdan SZAL Faculty of Mathematics, Computer Science and Econometrics, University of Zielona Góra, Zielona Góra POLAND
Brahimi Mahdi TAHAR Mathematiques et Informatique, Mathematiques, Mohamed Boudiaf University, Msila, ALGERIA
Jarno TALPONEN Department of Physics and Mathematics, University of Eastern Finland, Joensuu, FINLAND
Jakub TOMASZEWSKI Institute of Mathematics, Faculty of Electrical Engineering, Poznań University of Technology, Poznań, POLAND
Hans TRIEBEL Mathematics, Faculty of Mathematics and Computer Sciences, University of Jena, Jena, GERMANY
Marek WISŁA Faculty of Mathematics and Computer Science, Adam Mickiewicz University in Poznań, Poznań, POLAND

Michał WOJCIECHOWSKI IMPAN, Warszawa, POLAND
Przemysław WOJTASZCZYK Institute of Mathematics, Polish Academy of Sciences, Warszawa, POLAND
Paweł WÓJCIK Institute of Mathematics, Faculty of Mathematics, Physics and Technical Science, Pedagogical University of Cracow, Kraków, POLAND
Marek WÓJTOWICZ Mathematics, Mathematics, Physics and Techiques, Kazimierz Wielki University in Bydgoszcz, Bydgoszcz, POLAND
Damla YAMAN Mathematics/Faculty of Arts and Sciences, Yildiz Technical University, Istanbul, TURKEY
Elroy ZEEKOEI NWU, Mathematics and Statistics, North-West University, Potchefstroom, SOUTH AFRICA
Jing ZHOU School of Mathematics, Physics and Statistics, Shanghai University of Engineering Science, Shang Hai, CHINA